20-01-2022

Calcolare

$$\int \frac{dx}{2x^2-5x-3}$$

Calcolare

$$\int_1^e \frac{dx}{x(2\ln^2 x-5\ln x-3)}$$

Sia

$$\int \frac{dx}{2x^2-5x-3}$$

$$\Delta = 25-(2).(-3)(4) = 25+24 = 49 > 0$$

In generale

$$ax^2 + bx + c = 0 \qquad a \neq 1 \qquad \boxed{\Delta > 0} \quad (1)$$

e x_1, x_2 soluzioni distinte di (1)

$$ax^2 + bx + c = \underline{\underline{a}}(x - x_1)(x - x_2)$$

Nel nostro caso

$$2x^2 - 5x - 3 = 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25+24}}{4} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4} \begin{array}{l} 3 \\ -\frac{1}{2} \end{array}$$

Allora

$$2x^2 - 5x - 3 = 2(x-3)\left(x - \left(-\frac{1}{2}\right)\right) =$$

$$= 2(x-3)\left(x+\frac{1}{2}\right) = (x-3)(2x+1)$$

$$\frac{1}{2x^2-5x-3} = \frac{A}{x-3} + \frac{B}{2x+1} = \frac{A(2x+1)+B(x-3)}{2x^2-5x-3}$$

$$= \frac{2Ax+A+Bx-3B}{2x^2-5x-3} = \frac{x(2A+B)+A-3B}{2x^2-5x-3}$$

$$\iff \quad 1 = x(2A+B)+A-3B$$

$$\begin{cases} 2A+B=0 \\ A-3B=1 \end{cases} \qquad \begin{cases} B=-2A \\ A+6A=1 \end{cases} \qquad \begin{cases} B=-\frac{2}{7} \\ A=\frac{1}{7} \end{cases}$$

Allora

$$\frac{1}{2x^2-5x-3} = \frac{1}{7}\cdot\frac{1}{x-3} - \frac{2}{7}\,\frac{1}{2x+1}$$

Da ciò

$$\int \frac{1}{2x^2-5x-3}\,dx = \frac{1}{7}\int\frac{1}{x-3}\,dx - \frac{2}{7}\int\frac{1}{2x+1}\,dx$$

$$= \frac{1}{7}\ln|x-3| - \frac{1}{7}\int\frac{2}{2x+1}\,dx$$

$$\boxed{\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+c}$$

$$= \frac{1}{7}\ln|x-3| - \frac{1}{7}\ln|2x+1|+c$$

$$\boxed{D(2x+1)=2}$$

$$\int_1^e \frac{1}{x(\ln^2 x - 5\ln x - 3)}\,dx$$

dt

$$t = \ln x$$

$$dt = \frac{1}{x}\,dx$$

$$x = 1 \Rightarrow t = \ln 1 = 0$$

$$x = e \Rightarrow t = \ln e = 1$$

$$= \int_0^1 \frac{1}{t^2 - 5t - 3}\,dt =$$

$$= \left[\frac{1}{7}\ln|t-3| - \frac{1}{7}\ln|2t+1|\right]_0^1$$

$$= \underbrace{\frac{1}{7}\ln|-2| - \frac{1}{7}\ln|3|}_{t=1} - \underbrace{\frac{1}{7}\ln|-3| + \cancel{\frac{1}{7}\ln 1}}_{t=0}$$

$$= \frac{1}{7}\ln 2 - \frac{1}{7}\ln 3 - \frac{1}{7}\ln 3 = \frac{1}{7}(\ln 2 - 2\ln 3)$$

20 - 01 - 23

Calcolare

$$\int \frac{2x-3}{3x^2+5x-2}\,dx \quad e \quad \int_{-\frac{\pi}{2}}^{0} \frac{\cos x\,(2\sin x-3)}{3\sin^2 x+5\sin x-2}\,dx$$

$$\Delta = 25-(-2)(3)4 = 25+24 = 49$$

Troviamo le soluzioni di

$$3x^2+5x-2=0$$

$$x_{1,2} = \frac{-5\pm 7}{6} \quad \begin{matrix} -2 \\ \frac{1}{3} \end{matrix}$$

$$3x^2+5x-2 = 3(x+2)\left(x-\frac{1}{3}\right) = (x+2)(3x-1)$$

$$\frac{2x-3}{3x^2+5x-2} = \frac{A}{x+2} + \frac{B}{3x-1} =$$

$$= \frac{3Ax - A + Bx + 2B}{(x+2)(3x-1)} = \frac{x(3A+B) - A + 2B}{3x^2+5x-2}$$

$$\begin{cases} 3A+B = 2 \\ -A+2B = -3 \end{cases} \qquad \begin{cases} B = 2-3A \\ -A+2(2-3A) = -3 \end{cases}$$

$$\begin{cases} B = 2-3A \\ -A+4-6A = -3 \end{cases} \qquad \begin{cases} B = 2-3 = -1 \\ -7A = -7 \Rightarrow A = 1 \end{cases}$$

$$\frac{2x-3}{3x^2+5x-2} = \frac{1}{x+2} - \frac{1}{3x+1}$$

Da ciò

$$\int \frac{2x-3}{3x^2+5x-2}\,dx = \int \frac{1}{x+2}\,dx - \frac{1}{3}\int \frac{3 \cdot 1}{3x+1}\,dx$$

$$= \ln|x+2| - \frac{1}{3}\ln|3x+1| + c$$

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c}$$

Calcoliamo

$$\int_{-\frac{\pi}{2}}^{0} \frac{\cos x\,(2\sin x - 3)}{3\sin^2 x + 5\sin x - 2}\,dx \quad (*)$$

Si svolge per sostituzione

$$t = \sin x \Rightarrow dt = \cos x \, dx$$

$$x = -\frac{\pi}{2} \Rightarrow t = \sin\left(-\frac{\pi}{2}\right) = -1$$

$$x = 0 \Rightarrow t = \sin 0 = 0$$

Allora

$$(*) \quad \int_{-1}^{0} \frac{2t-3}{3t^2+5t-2} \, dt =$$

$$= \Big[\ln|t+2| - \frac{1}{3} \ln|3t-1| \Big]_{-1}^{0} =$$

$$= \underbrace{\ln 2 - \overset{0}{\overset{\parallel}{\cancel{\tfrac{1}{3}\ln 1}}}}_{t=0} - \underbrace{\overset{0}{\overset{\parallel}{\cancel{\ln 1}}} + \frac{1}{3}\ln 4}_{t=-1}$$

06-02-23

$$\int \frac{3x-1}{x^2+6x+9}\,dx \quad e \quad \int_{e^{-1}}^{e} \frac{(3\ln x-1)}{x(\ln^2 x+6\ln x+9)}\,dx$$

$$\Delta = 36-4\cdot 1\cdot 9 = 36-36 = 0$$

$$x^2+6x+9 = (x+3)^2$$

Inquanto la scomposizione è

$$\frac{3x-1}{x^2+6x+9} = \frac{A}{x+3} + \frac{B}{(x+3)^2} = \frac{Ax+3A+B}{(x+3)^3}$$

$$\begin{cases} A = 3 \\ 3A + B = -1 \end{cases} \qquad \begin{cases} A = 3 \\ 9 + B = -1 \end{cases} \qquad \begin{cases} A = 3 \\ B = -10 \end{cases}$$

Da ciò

$$\frac{3x-1}{x^2+6x+9} = \frac{3}{x+3} - \frac{10}{(x+3)^2}$$

e

$$\int \frac{3x-1}{x^2+6x+9}\,dx = 3\int \frac{1}{x+3}\,dx - 10\int \frac{1}{(x+3)^2}\,dx$$

$\Downarrow$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int \frac{f'(x)}{f(x)^2}\,dx = \frac{f(x)^{-2+1}}{-2+1} + c = -\frac{1}{f(x)} + c$$

$$\int f(x)^{\alpha} f'(x)\,dx = \frac{f(x)^{\alpha+1}}{\alpha+1} + c \qquad \alpha \neq -1$$

Nel nostro caso $\underline{\alpha = -2}$

Allora

$$3\int \frac{1}{x+3} - 10\int \frac{1}{(x+3)^2}\,dx = 3\ln|x+3|$$

$$-10\left[-\frac{1}{x+3}\right] + c =$$

$$= 3\ln|x+3| + \frac{10}{x+3} + c$$

Calcolo

$$\int_{e^{-1}}^{e} \frac{3\ln x - 1}{x(\ln^2 x + 6\ln x + 9)}\, dx$$

$\ln x = t$

$dt = \frac{1}{x}\, dx$

$$= \int_{-1}^{1} \frac{3t - 1}{t^2 + 6t + 9}\, dx$$

$x = e^{-1} \Rightarrow t = \ln e^{-1} =$

$= -1 \ln e = -1$

$x = e \Rightarrow t = \ln e = 1$

$$= \left[3 \ln |t+3| - \frac{10}{t+3} \right]_{-1}^{1}$$

$= - \_\_ -$

$$\int \frac{x}{x^2+4x+8}\,dx \quad e \quad \int_{-\sqrt[5]{4}}^{0} \frac{x^9}{x^{10}+4x^5+8}\,dx$$

Cominciamo con risolvere

$$\int \frac{x}{x^2+4x+8}\,dx$$

$$\Delta = (4)^2 - 4 \cdot 1 \cdot 8 = 16 - 32 = -16$$

$$\int \frac{f'(x)}{[f(x)]^2+1}\,dx = \arctan f(x) + c$$

$$x^2+4x+8 = x^2+4x+4+4 = (x+2)^2+4 =$$

$$= 4\left[\frac{(x+2)^2}{4}+1\right] = 4\left[\left(\frac{x+2}{2}\right)^2+1\right]$$

Col dieu

$$\int \frac{x}{x^2+4x+8}\,dx = \frac{1}{2}\int \frac{2x+4-4}{x^2+4x+8}\,dx =$$

$$= \frac{1}{2}\int \frac{2x+4}{x^2+4x+8}\,dx - \frac{1}{\cancel{2}}\cdot\overset{2}{\cancel{4}}\int \frac{1}{x^2+4x+8}\,dx$$

$$= \frac{1}{2}\ln\left(x^2+4x+8\right) - 2\int \frac{1}{x^2+4x+8}\,dx =$$

$$= \frac{1}{2}\ln(x^2+4x+8) - 2\int \frac{1}{4\left[\left(\frac{x+2}{2}\right)^2+1\right]}\,dx =$$

$$= \frac{1}{2}\ln(x^2+4x+8) - \frac{1}{2}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx =$$

$$= \frac{1}{2}\ln(x^2+4x+8) - \int \frac{\frac{1}{2}}{\left(\frac{x+2}{2}\right)^2+1}\,dx =$$

$$= \frac{1}{2}\ln(x^2+4x+8) - \arctan\left(\frac{x+2}{2}\right) + c$$

~ ~

$$\int_{-\sqrt[5]{4}}^{0} \frac{x^9}{x^{10}+4x^5+8}\,dx \qquad x^9 = x^4 \cdot x^5 \qquad \Bigg| \begin{array}{l} x^5 = t \\ dt = 5x^4\,dx \end{array}$$

$$= \frac{1}{5}\int_{-\sqrt[5]{4}}^{0} \frac{5x^4 \cdot x^5}{x^{10}+4x^5+8}\,dx$$

$$x^5 = t$$

$$dt = 5x^4\,dx$$

$$x = 0 \Rightarrow t = 0$$

$$x = -\sqrt[5]{4} \Rightarrow$$

$$\Rightarrow t = \left(-\sqrt[5]{4}\right)^5 = -4$$

$$= \frac{1}{5}\int_{-4}^{0} \frac{t}{t^2+4t+8}\,dt$$

$$= - - - -$$

Risolvere

$$\underbrace{\int \frac{x}{2x^2-x-1}\,dx}_{\downarrow} \quad e \quad \int_0^{\frac{1}{4}} \frac{1}{2x-\sqrt{x}-1}\,dx$$

$$2x^2-x-1 = 2(x-1)\left(x+\frac{1}{2}\right) = (x-1)(2x+1)$$

FRATTI SEMPLICI

$$\frac{x}{2x^2-x-1} = \frac{A}{x-1} + \frac{B}{2x+1}$$

(continuiamo e cose)

$$\int_0^{\frac{1}{4}} \frac{dx}{2x-\sqrt{x}-1} =$$

$$= \int_0^{\frac{1}{2}} \frac{2t}{2t^2-t-1}\,dt =$$

$$= 2 \int_0^{\frac{1}{2}} \frac{2t}{2t^2-t-1}\,dt$$

$$\Downarrow$$

STESSO INTEGRALE DI PRIMA

Si svolge per sostituzione

$$t = \sqrt{x} \Rightarrow x = t^2$$

$$dx = 2t\,dt$$

$$x = 0 \Rightarrow t = \sqrt{0} = 0$$

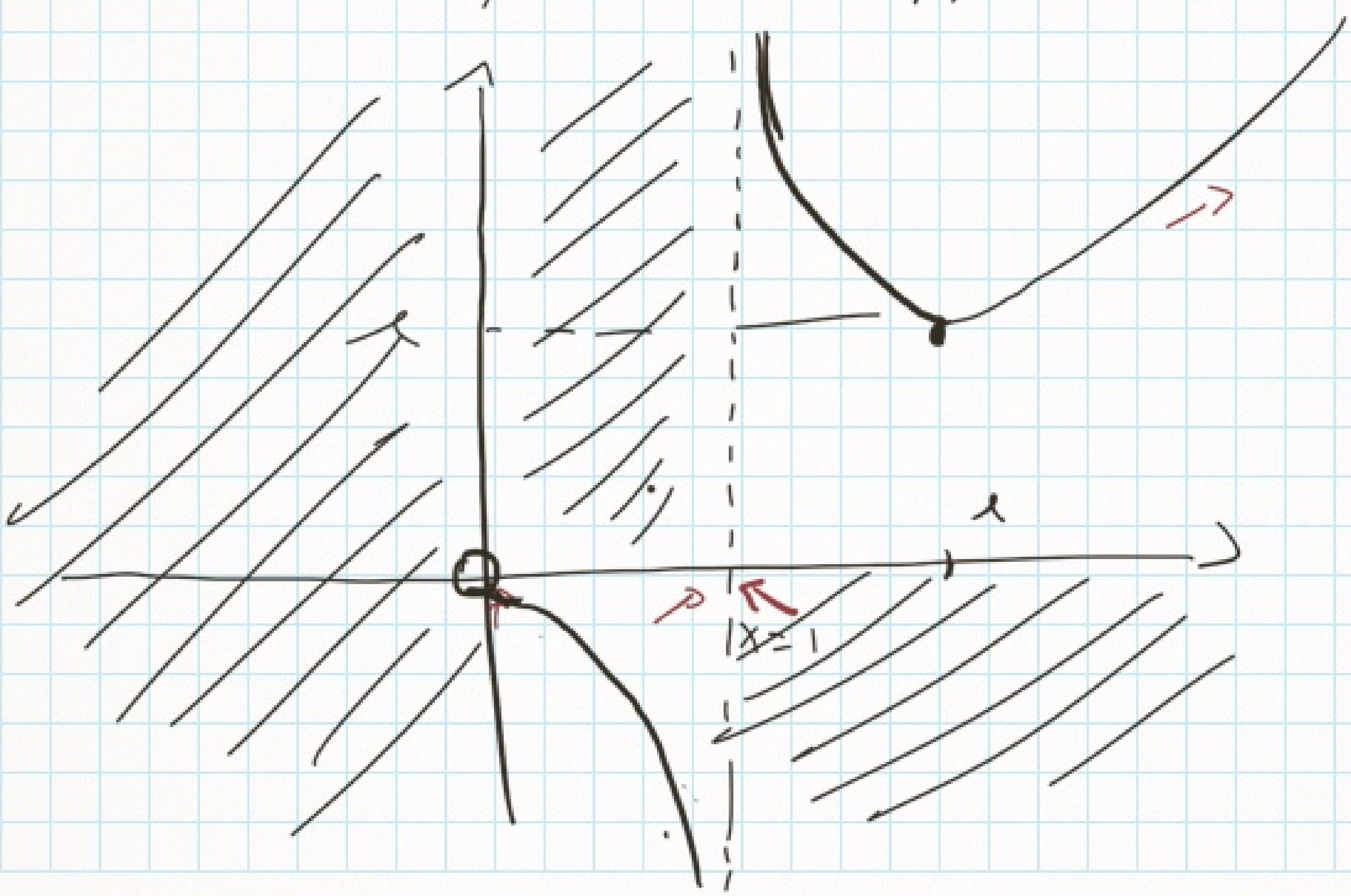
$$x = \frac{1}{4} \Rightarrow t = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Studiare il grafico di

$$f(x) = \frac{x}{\ln x}$$

Dominio: $x > 0$ (dominio della funzione log)

$\ln x \neq 0 \Rightarrow x \neq 1$



$x=0$ non è nel dominio, quindi non ci sono intersezioni con l'asse delle y

$$f'(x)=0 \Rightarrow \frac{x}{\ln x}=0 \iff x=0$$

$\Downarrow$

non è elemento del dominio!

non ci sono intersezioni con l'asse delle x

$$f'(x)>0 \iff \frac{x}{\ln x}>0$$

$x>0$ nel dominio

$$\iff \ln x>0 \iff x>1$$

Asintoti

$$\lim_{x\to 0^+} \frac{x}{\ln x} = \frac{0}{\ln 0^+} = \frac{0}{-\infty} = 0$$

f non ha un asintoto verticale in $x=0$.

$$\lim_{x\to 1^-} \frac{x}{\ln x} = \lim_{x\to 1^-} \frac{1}{\ln 1} = \lim_{x\to 1^-} \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 1^+} \frac{x}{\ln x} = +\infty$$

(guardare la positività della funzione ($f(x) \geq 0$))

$$\lim_{x\to +\infty} \frac{x}{\ln x} = +\infty$$

(x va a $+\infty$ più velocemente di $\ln x$)

non c'è asintoto orizzontale

Asintoto obliquo

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x}{\frac{\ln x}{x}} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\ln x} = \lim_{x\to+\infty} \frac{1}{\ln x} = \frac{1}{+\infty} = 0$$

Allora non esiste l'asintoto obliquo ("$m \neq 0$")

Studiamo $f'(x)$

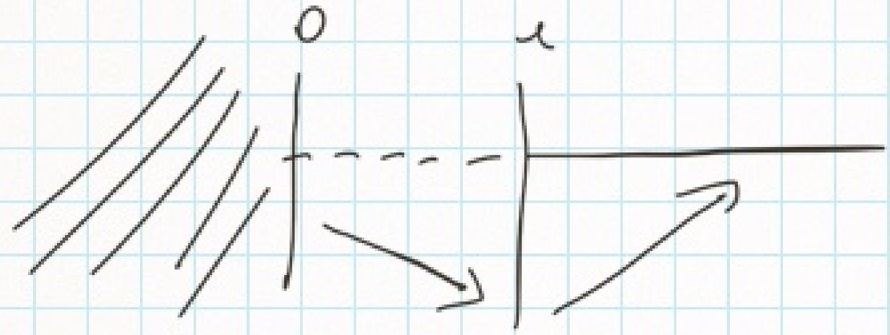
$$f(x) = \frac{x}{\ln x}$$

$$f'(x) = \frac{1\cdot \ln x - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$$

$f'(x) \geq 0$ (per trovare i punti di massimo e minimo relativo)

$$\frac{\ln x - 1}{\underbrace{\ln^2 x}_{\geq 0}} \geq 0 \iff \ln x - 1 \geq 0 \iff$$

$$\iff \ln x \geq 1 \iff x \geq e$$



$x = e$ punto di minimo relativo

$f(e) = e$